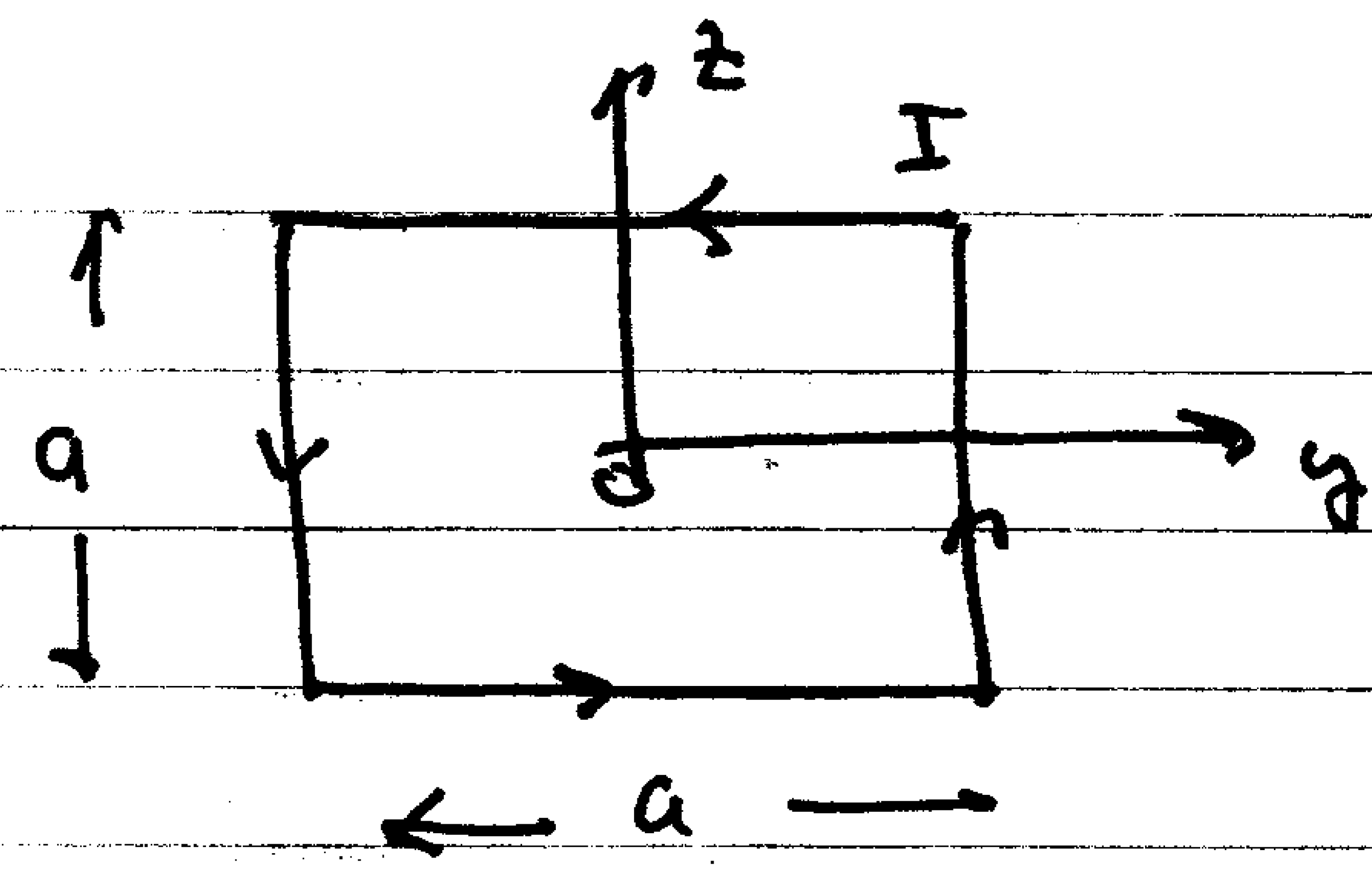
PHYS 321
Assignment 4
Due Monday, April 2, 2018

Read Chapters 5, 6

Problems of Chapter 5: 4, 6, 10a, 12, 14, 17, 23, 30, 37, 48

5-4)



$$\vec{B} = h z \hat{x} \quad \text{Note } \hat{x} = \odot$$

$$\vec{F} = I \vec{l} \times \vec{B}$$

$$\vec{F}_{top} = I h \left(\frac{a}{2}\right)(-\hat{y} \times \hat{x})\, a$$

$$= I h \frac{a^2}{2} \hat{z}$$

$$\vec{F}_{bottom} = I h \left(-\frac{a}{2}\right)(\hat{y} \times \hat{x})\, a$$

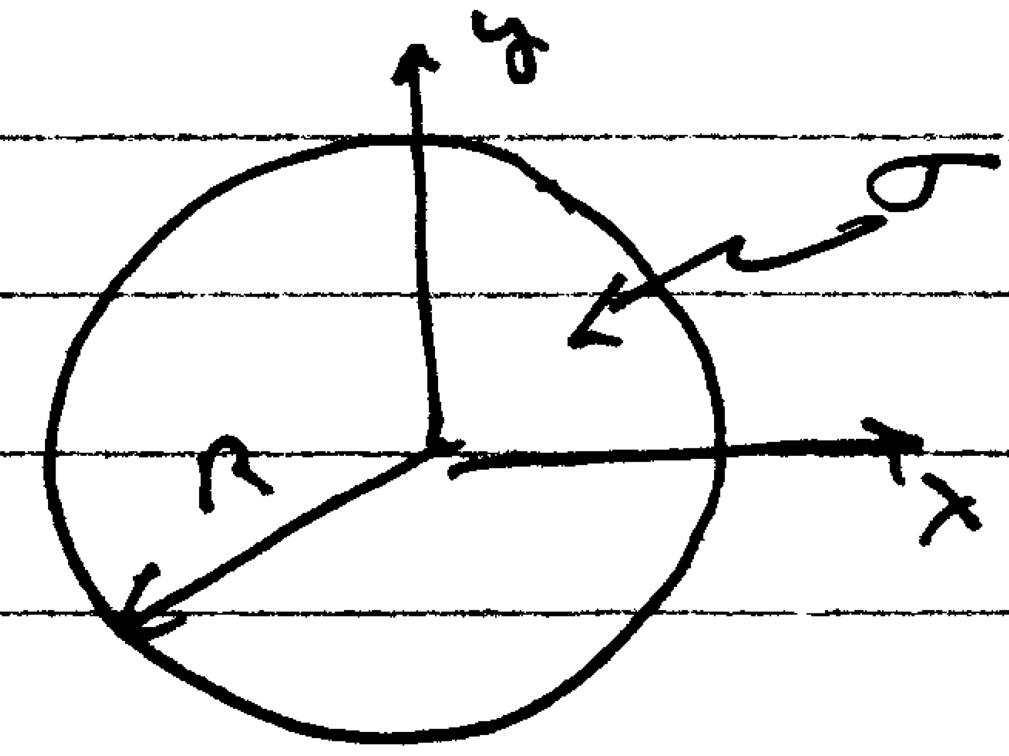
$$= I h \frac{a^2}{2} \hat{z}$$

Note: $\vec{F}_{left} = -\vec{F}_{right}$

$$\vec{F}_{total} = I h a^2 \hat{z}$$

5.6)

a)

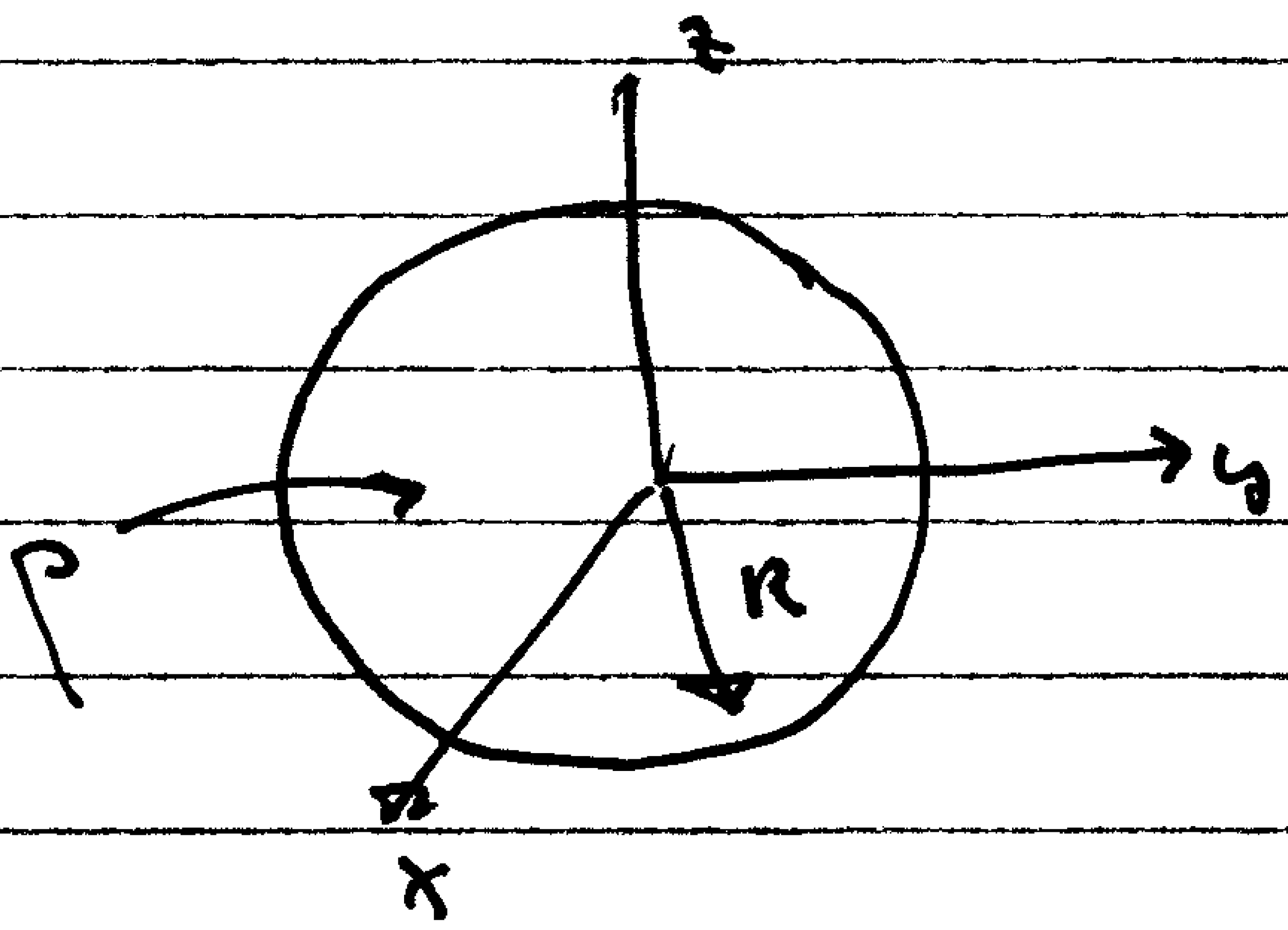


$\vec{K} = \sigma \vec{v}$

$\vec{v} = \omega r \hat{\varphi}$

$\vec{K} = \omega r \sigma \hat{\varphi}$

b)



$\vec{J} = \rho \vec{v}$

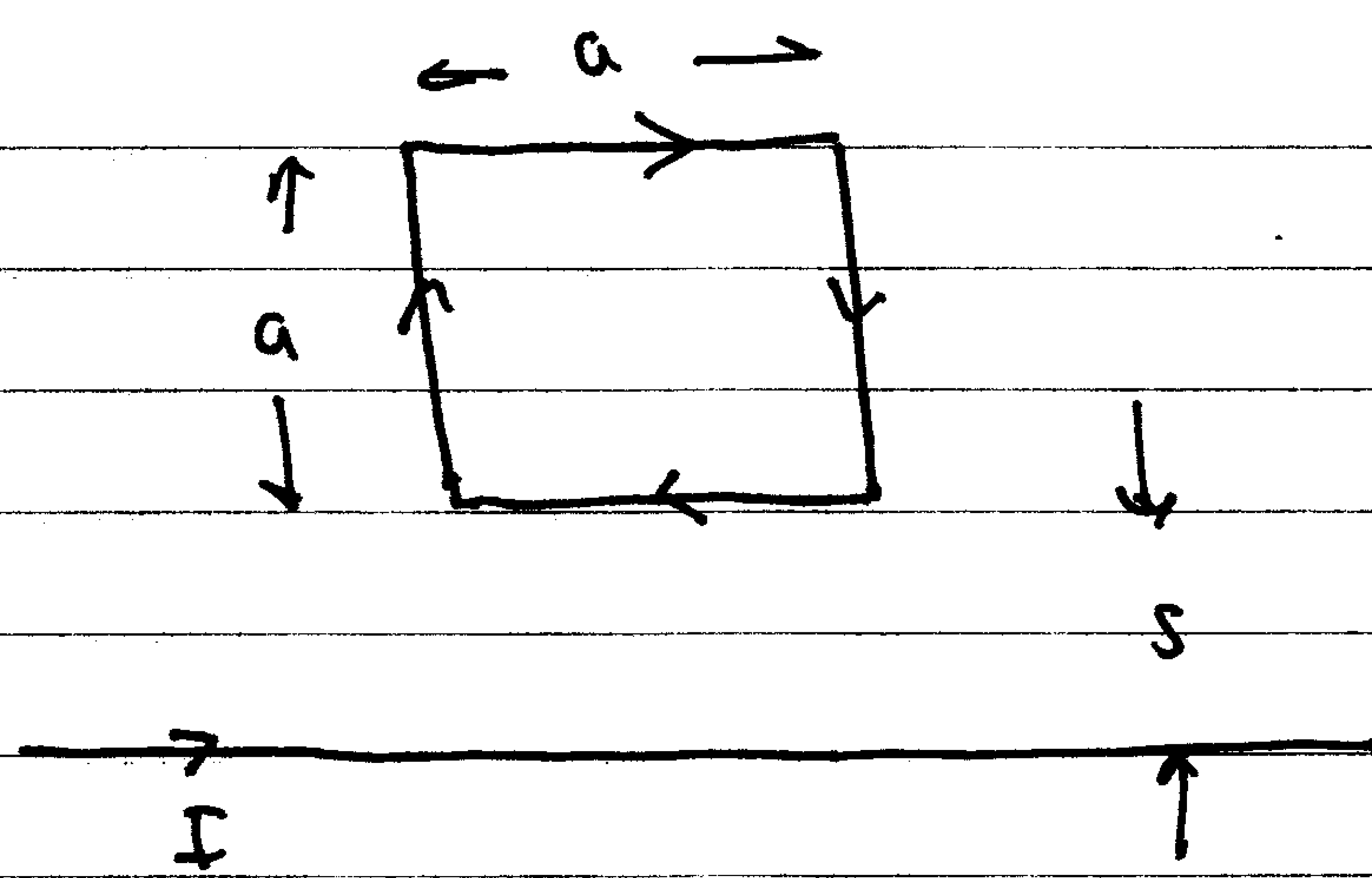
$\vec{v} = r \sin\theta \, \omega \hat{\varphi}$

$\vec{J} = \rho r \omega \sin\theta \, \hat{\varphi}$

Let $\rho = \dfrac{Q}{\frac{4\pi}{3} R^3}$

$\vec{J} = \dfrac{3 Q r \omega \sin\theta}{4\pi R^3} \hat{\varphi}$

10a)



$\vec{B}$ due to the long wire is

$$B = \frac{\mu_0 I}{2\pi s} \odot \text{ above the wire}$$

$$\vec{F} = I\,\vec{l} \times \vec{B} \qquad \vec{F}_{top} = \left(\frac{\mu_0 I}{2\pi(s+a)}\right)(I a) \downarrow$$

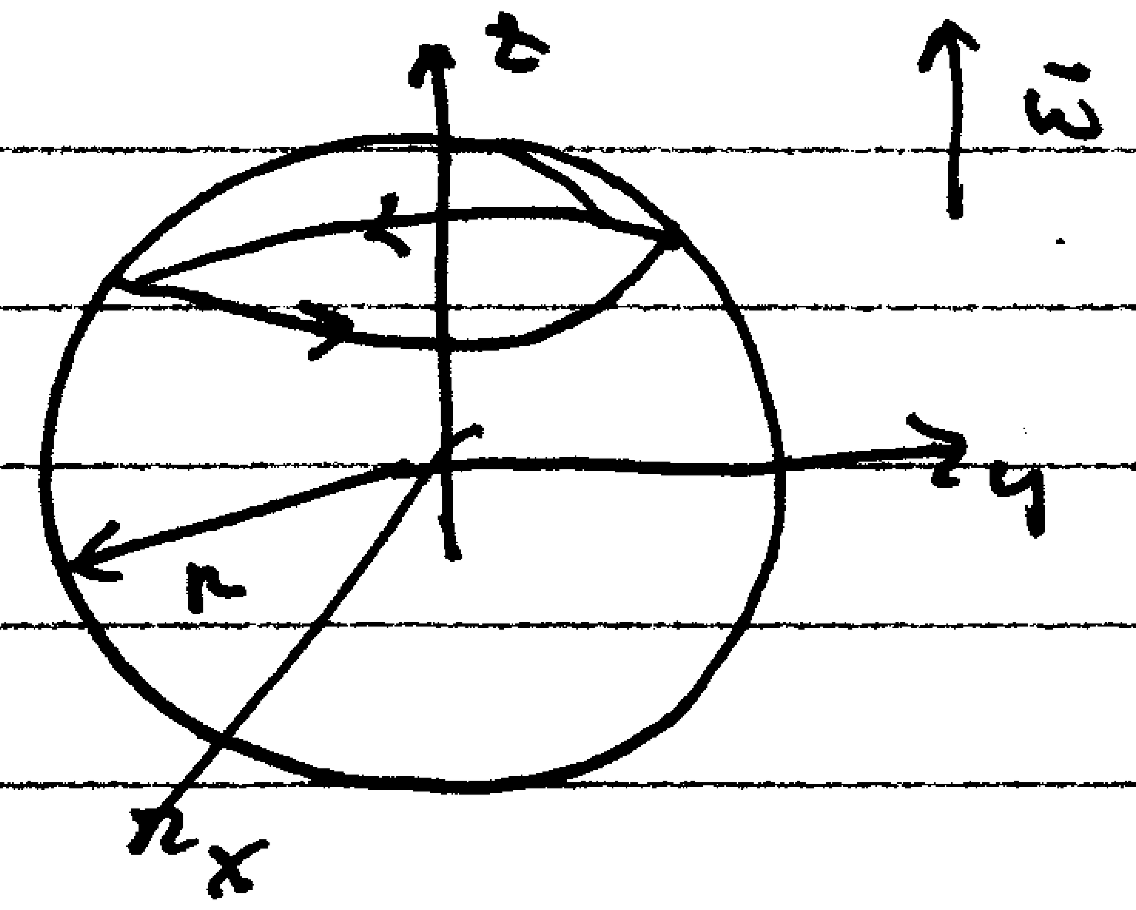
$$\vec{F}_{top} = \frac{\mu_0 I^2 a}{2\pi(s+a)} \downarrow$$

$$\vec{F}_{bottom} = \frac{\mu_0 I^2 a}{2\pi s} \uparrow$$

$$\vec{F}_{total} = \frac{\mu_0 I^2 a}{2\pi}\left(\frac{1}{s} - \frac{1}{s+a}\right) = \frac{\mu_0 I^2 a}{2\pi}\,\frac{a}{s(s+a)} \uparrow$$

Note $\vec{F}_{left} = -\vec{F}_{right}$ and cancel

5.12)



At a polar angle θ, $r = R\sin\theta$ $v = \omega R \sin\theta$

so $K = \sigma v = \sigma \omega R \sin\theta$

$$dI = K\,R\,d\theta = \sigma \omega R^2 \sin\theta\, d\theta$$

From Equ 5.41, we have for loop

$$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$$

Let $I \to dI = \sigma\omega R^2 \sin\theta\, d\theta$

$R \to r = R\sin\theta$

$z \to R\cos\theta$

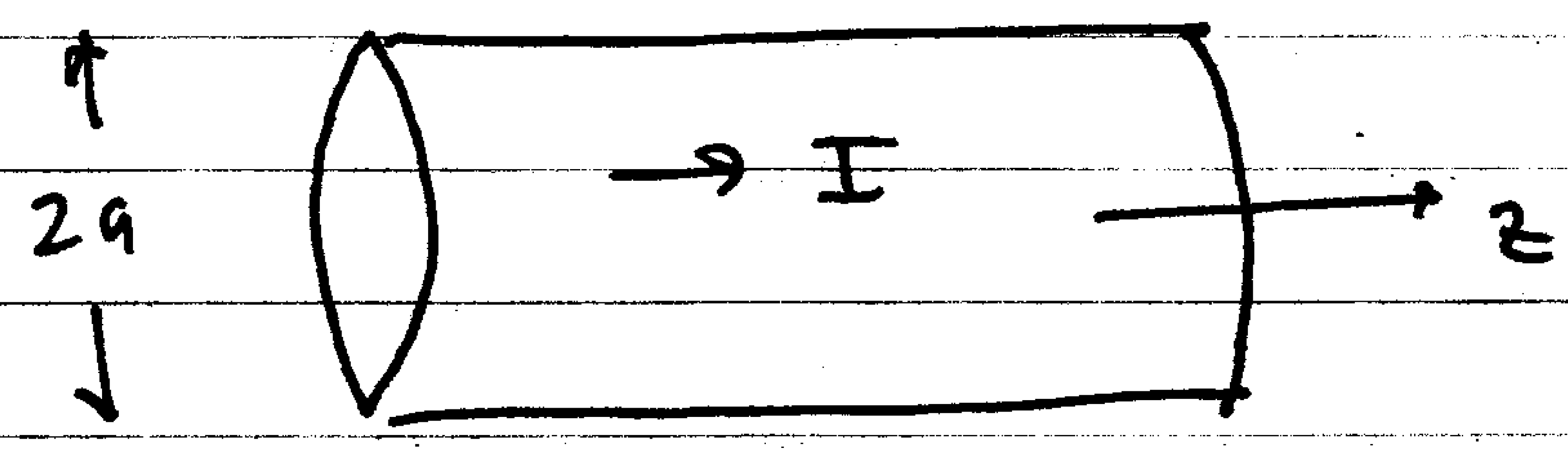
$$dB = \frac{\mu_0}{2} \frac{\sigma\omega R^2 \sin\theta\,(R^2\sin^2\theta)\,d\theta}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}$$

$$B = \frac{\mu_0}{2}\sigma\omega R \int_0^\pi d\theta \sin^3\theta$$

$$= \frac{\mu_0}{2}\sigma\omega R\left(\frac{4}{3}\right) = \frac{\mu_0 2}{3}\omega R \frac{Q}{4\pi R^2} = \frac{\mu_0 \omega Q}{6\pi R}$$

$$\vec{B} = \frac{\mu_0 Q}{6\pi R}\vec{\omega}$$

5.14)



a) Use Ampere $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

$$B = 0 \quad r < a$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\varphi} \quad s > a$$

b) $B(2\pi s) = \mu_0 \int \vec{J}\cdot d\vec{a} \qquad J = ks \qquad da = 2\pi s\, ds$

$$B(2\pi s) = \mu_0 k \int_0^s d\bar{s}\, 2\pi \bar{s}^2 = \mu_0 k (2\pi) \frac{s^3}{3}$$

$$\vec{B} = \frac{\mu_0 k s^2}{3}\hat{\varphi} \quad s < a$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\varphi} \quad s > a$$

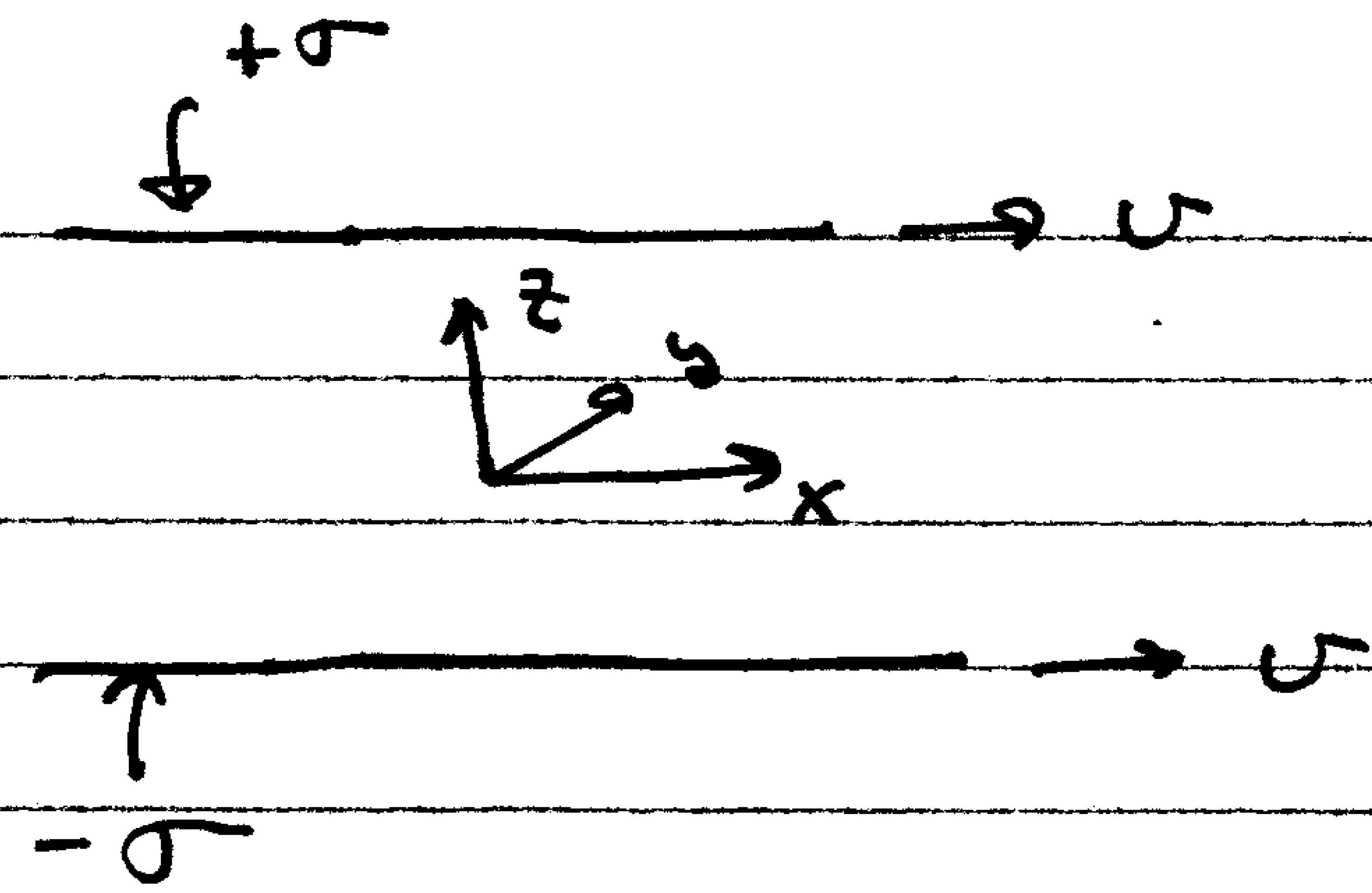
determine k $\quad I = \int \vec{J}\cdot d\vec{a} = \int_0^a (2\pi) s\, ds\, (ks)$

$$= 2\pi \int_0^a ds\, s^2 k = \frac{2\pi k a^3}{3}$$

$k = \frac{3I}{2\pi a^3}$ So for $s < a$, we have

$$\vec{B} = \frac{\mu_0 s^2}{3}\left(\frac{3I}{2\pi a^3}\right)\hat{\varphi} = \frac{\mu_0 I s^2}{2\pi a^3}\hat{\varphi}$$

5.17)



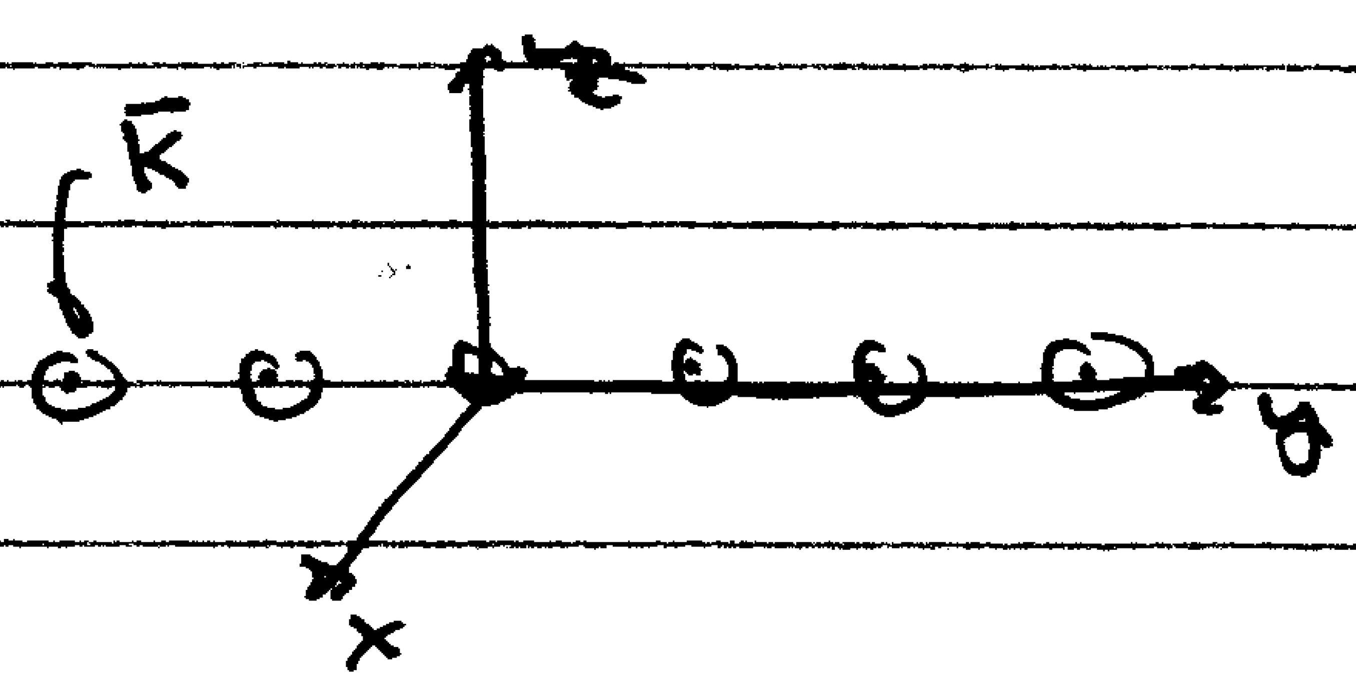
Note y is $\otimes$

a) For a single infinite sheet of charge

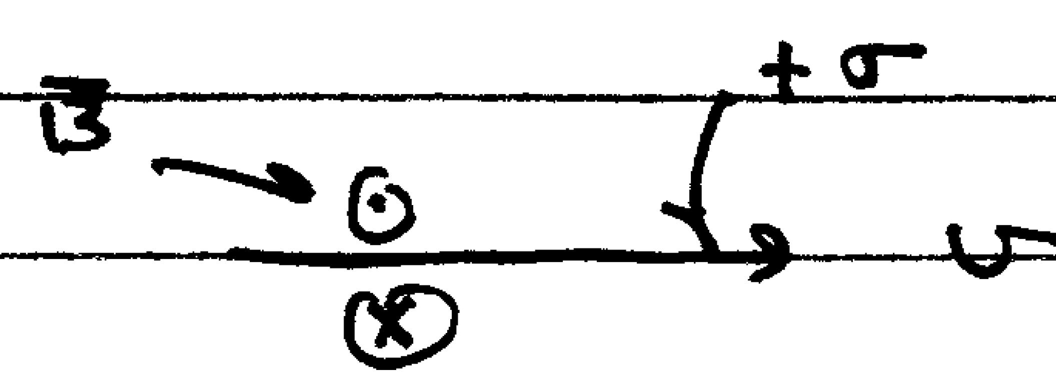
$$\bar{B} = \frac{\mu_0 K}{2}\hat{y} \qquad z<0$$

$$\bar{B} = -\frac{\mu_0 K}{2}\hat{y} \qquad z>0$$

see co-ordinate and diagram below.



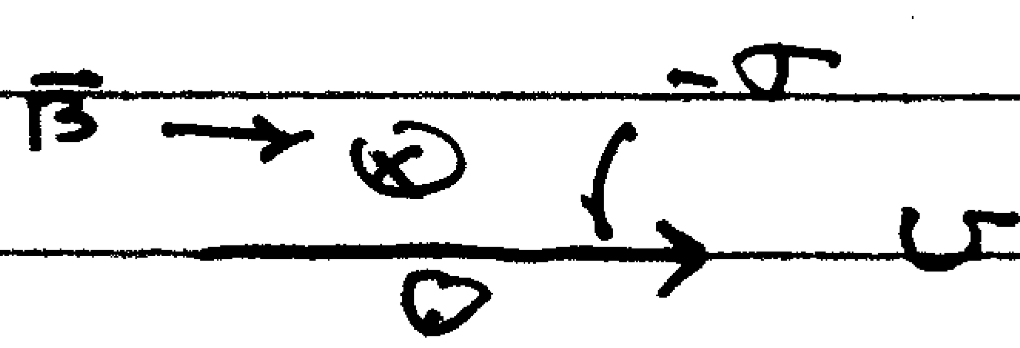
So for the top moving plate at $z>0$



$$B = \frac{\mu_0 K}{2} = \frac{\mu_0 \sigma v}{2}$$

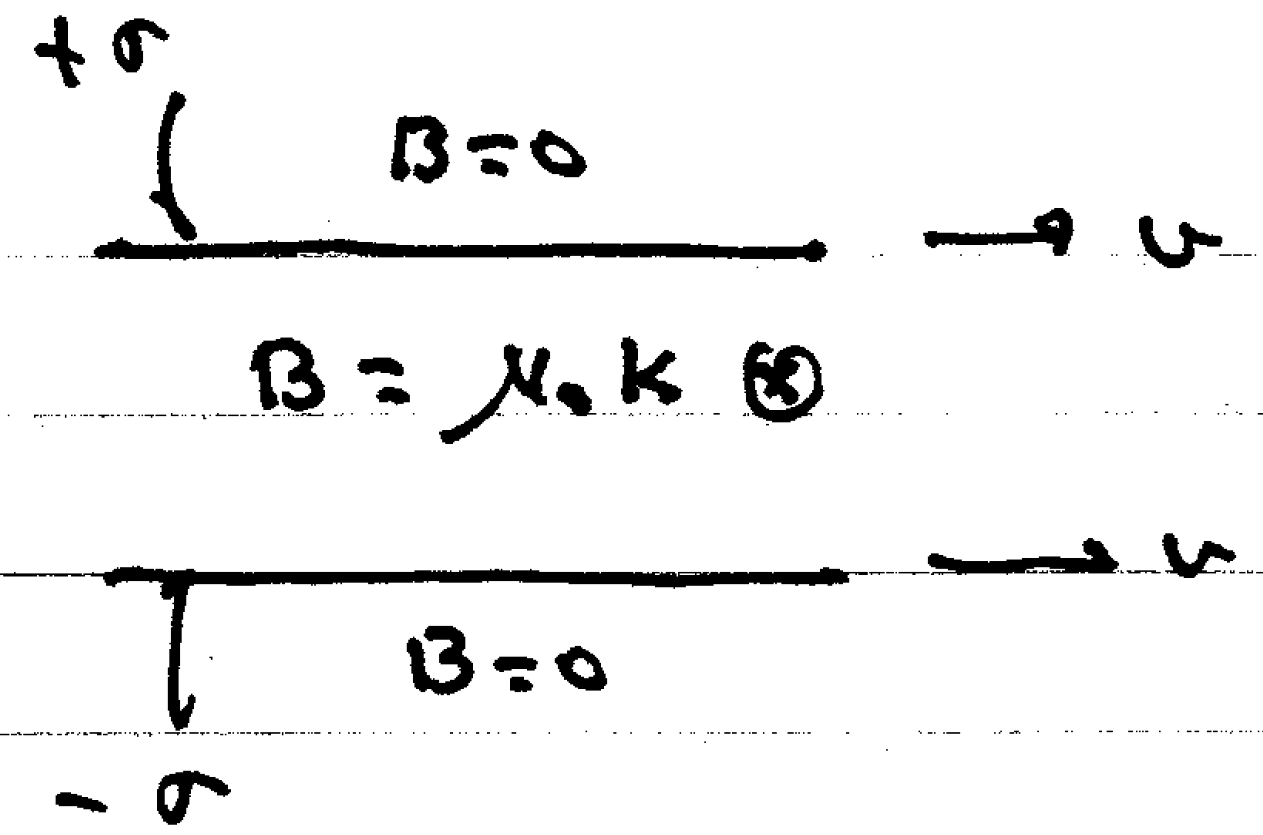
with $K = \sigma v$.

bottom moving plate at $z<0$



$$B = \frac{\mu_0 K}{2} = \frac{\mu \sigma v}{2}$$

Since B is independent of distance, the fields add between the plates and cancel above and below the plates

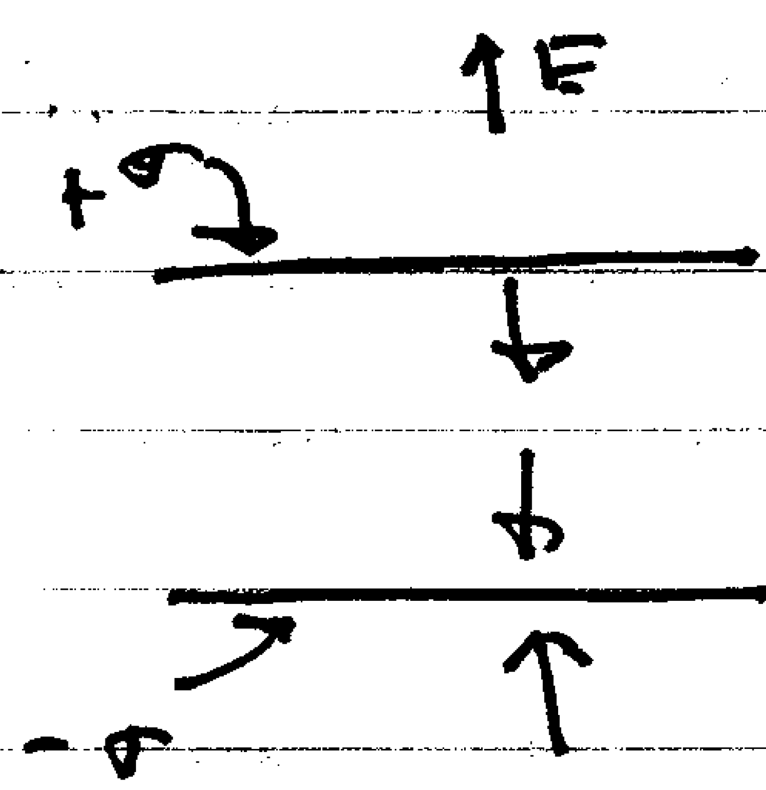


b) $\vec{F} = \vec{K} \times \vec{B}\, da$ $\qquad \dfrac{F}{A} = K\left(\dfrac{\mu_0 K}{2}\right) = \dfrac{\mu_0 \sigma^2 v^2}{2} \uparrow$

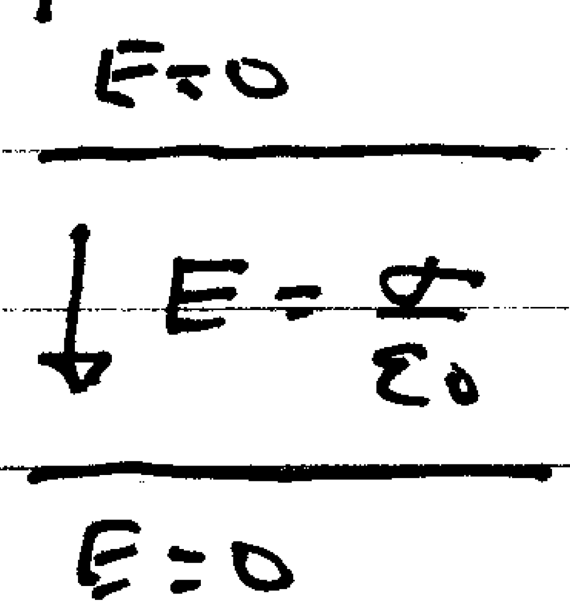
$=$ force on upper plate

c) The electric fields are given by

$E = \dfrac{\sigma}{2\varepsilon_0}$



the electric fields cancel above and below the plates and add between the plates

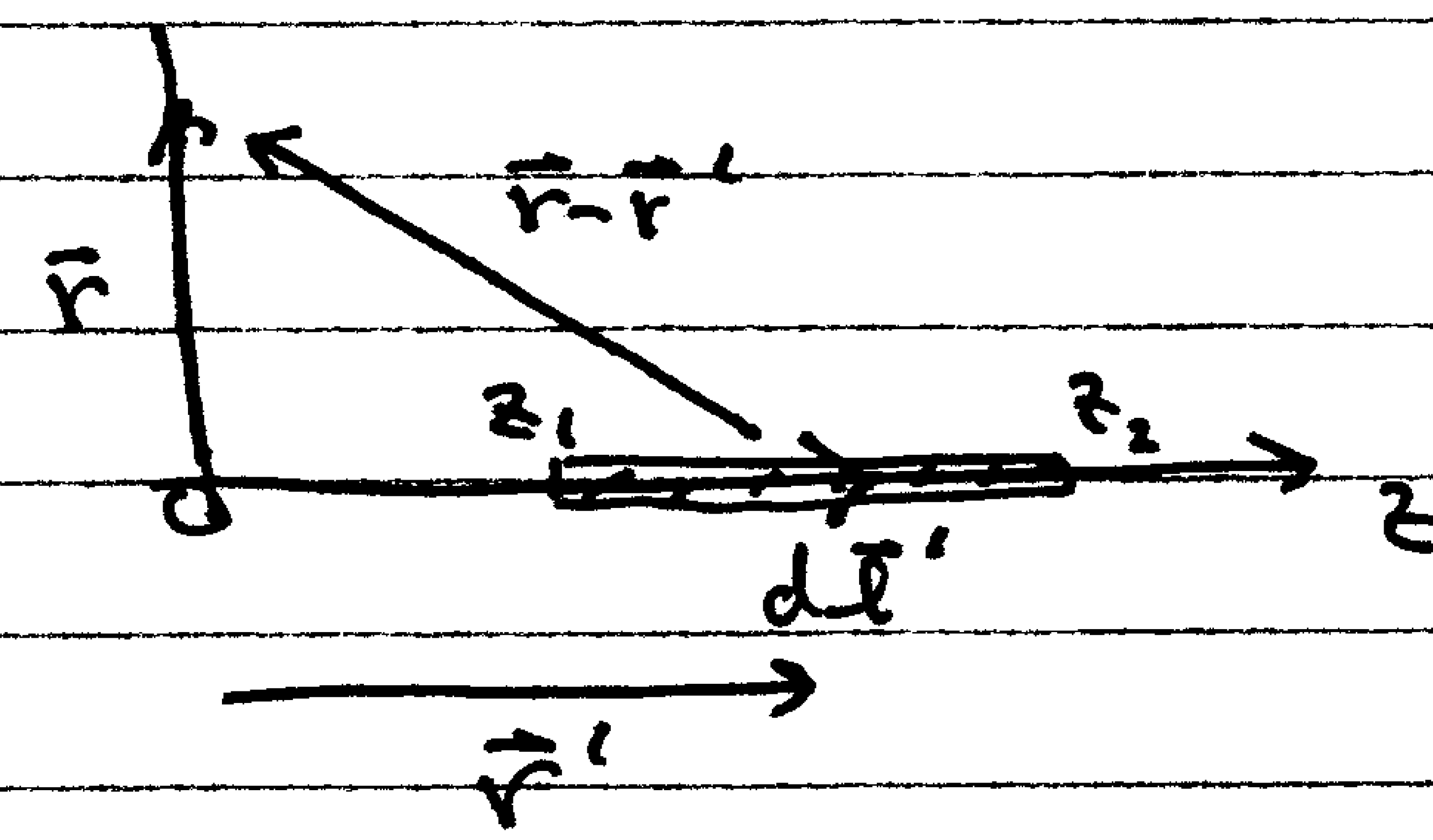


the $\dfrac{\text{force}}{\text{area}}$ on the top plate due to the bottom plate is $\dfrac{F}{A} = \dfrac{\sigma^2}{2\varepsilon_0} \downarrow$

the magnetic and electric forces balance when

$\dfrac{\sigma^2}{2\varepsilon_0} = \dfrac{\mu_0 \sigma^2 v^2}{2} \qquad v = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$, speed of light

5.23) $\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}'}{|\vec{r}-\vec{r}'|}$



$\vec{r} = s\,\hat{s}$

$\vec{r}' = z'\,\hat{z}$

$|\vec{r}-\vec{r}'| = (s^2+z'^2)^{1/2}$

$d\vec{l}' = dz'\,\hat{z}$

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\,\hat{z} \int_{z_1}^{z_2} \frac{dz'}{(s^2+z'^2)^{1/2}}$$

$$= \frac{\mu_0 I}{4\pi}\,\hat{z}\, \ln\left[z' + (z'^2+s^2)^{1/2}\right] \Big|_{z'=z_1}^{z'=z_2}$$

$$= \frac{\mu_0 I}{4\pi}\,\hat{z}\, \ln\left(\frac{z_2 + (z_2^2+s^2)^{1/2}}{z_1 + (z_1^2+s^2)^{1/2}}\right)$$

$$\vec{B} = \nabla \times \vec{A} = -\frac{\partial A_z}{\partial s}\,\hat{\varphi}$$

$$A_z(s) = \frac{\mu_0 I}{4\pi}\left[\ln\left(z_2 + (z_2^2+s^2)^{1/2}\right) - \ln\left(z_1 + (z_1^2+s^2)^{1/2}\right)\right]$$

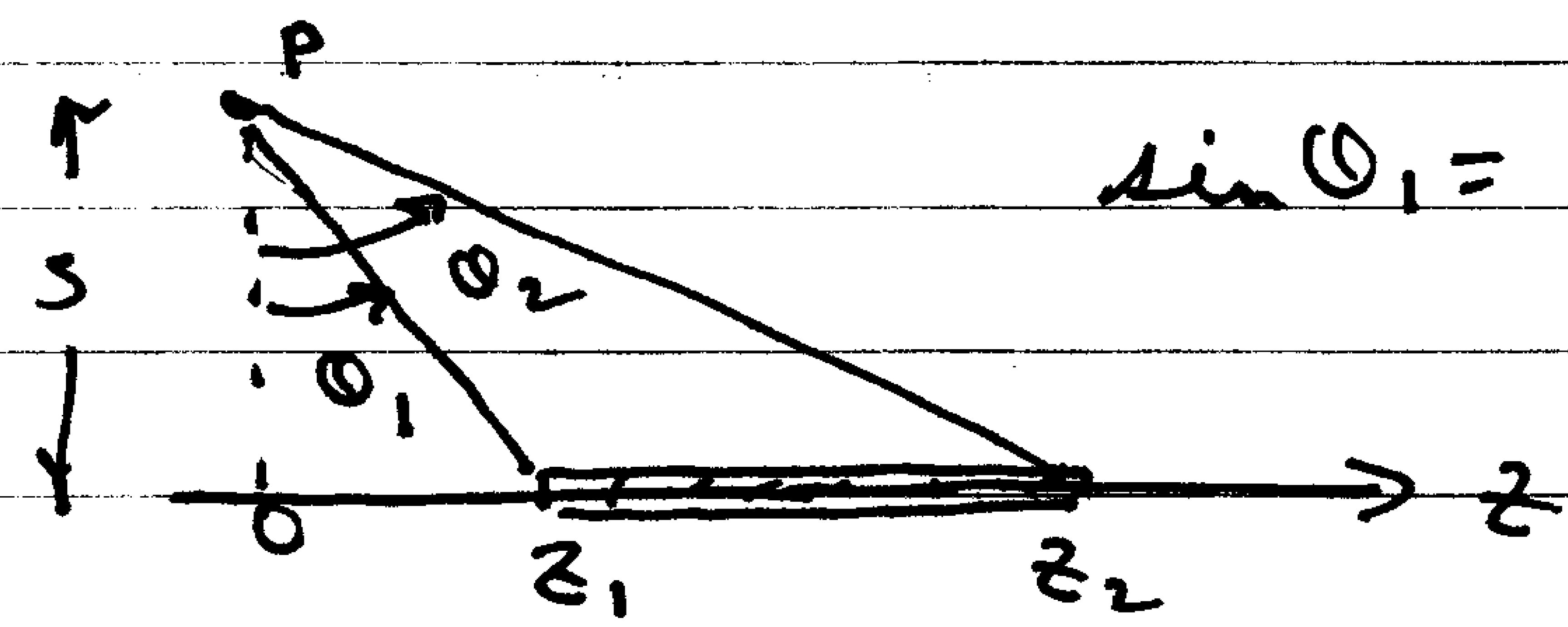
$$\frac{\partial A_z}{\partial s} = \frac{\mu_0 I}{4\pi}\left[\frac{\frac{1}{2}(z_2^2+s^2)^{-1/2}\,2s}{z_2+(z_2^2+s^2)^{1/2}} - \frac{\frac{1}{2}(z_1^2+s_2^2)^{-1/2}\,2s}{z_1+(z_1^2+s^2)^{1/2}}\right]$$

$$= \frac{\mu_0 I s}{4\pi}\left[\frac{z_2-(z_2^2+s^2)^{1/2}}{(z_2^2+s^2)^{1/2}(z_2^2-z_2^2-s^2)} - (z_2 \to z_1)\right]$$

$$= -\frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{(z_2^2+s^2)^{1/2}} - 1 - \frac{z_1}{(z_1^2+s^2)^{1/2}} + 1\right]$$

$$= -\frac{\mu_0 I}{4\pi s}\left(\frac{z_2}{(z_2^2+s^2)^{1/2}} - \frac{z_1}{(z_1^2+s^2)^{1/2}}\right)$$

$$\vec{B} = \frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{(z_2^2+s^2)^{1/2}} - \frac{z_1}{(z_1^2+s^2)^{1/2}}\right]\hat{\varphi}$$



$$\sin\theta_1 = \frac{z_1}{\sqrt{z_1^2+s^2}}$$

$$\sin\theta_2 = \frac{z_2}{(z_2^2+s^2)^{1/2}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right)\hat{\varphi}$$

Note $\theta_2 \to \pi/2$ $\theta_1 \to -\pi/2$

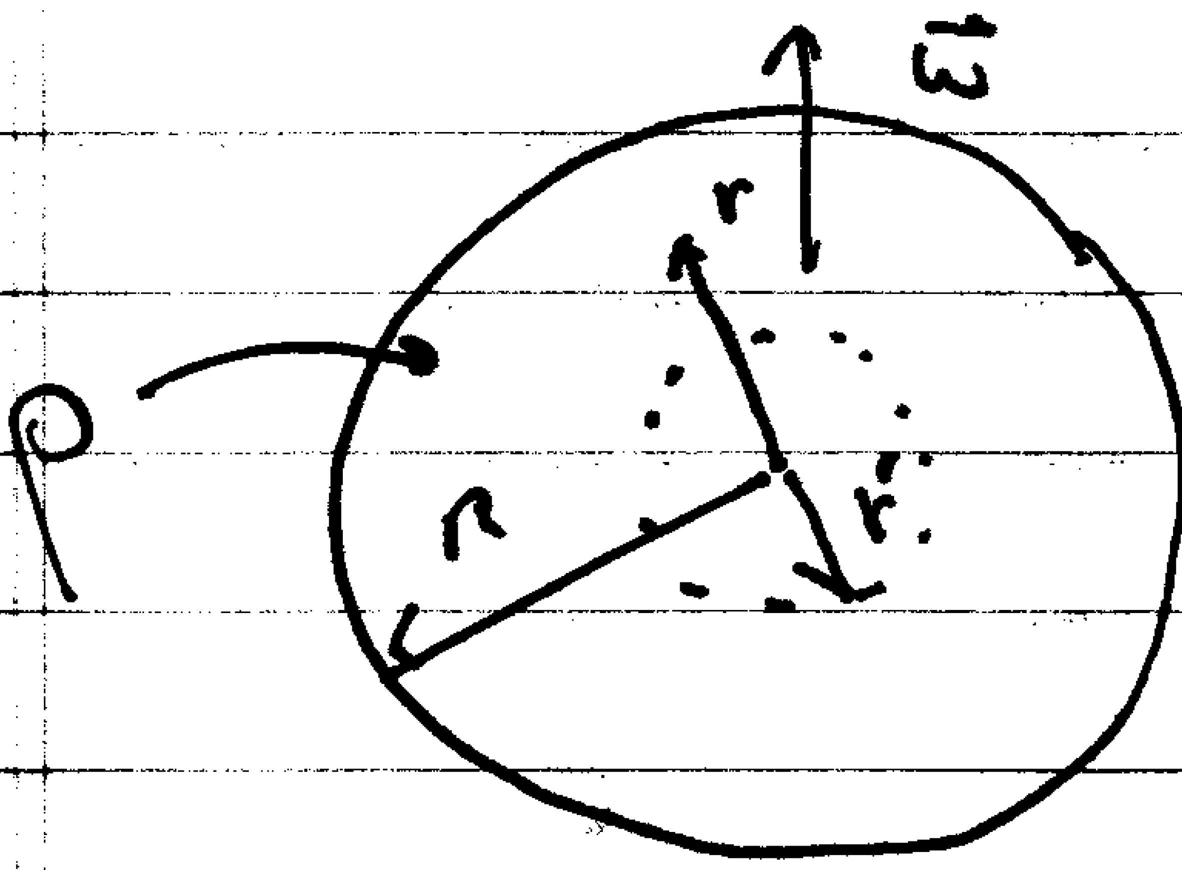
$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\varphi}$ the result for an infinite wire

5.30) For a spinning shell

$$\vec{A} = \frac{\mu_0 R \omega \sigma r \sin\theta}{3}\,\hat{\varphi} \qquad r \le R$$

$$\vec{A} = \frac{\mu_0 R^4 \omega \sigma}{3}\,\frac{\sin\theta}{r^2}\,\hat{\varphi} \qquad r \ge R$$

Break up the solid spinning sphere into differential shells of thickness dr' and radius r'



Note: $4\pi r^2 \rho\, dr = 4\pi r^2 \sigma$
so $\sigma \rightarrow \rho\, dr$

When $r' < r$, we have a contribution

$$dA = \frac{\mu_0 (r')^4 \omega \rho\, dr'}{3}\,\frac{\sin\theta}{r^2} \qquad r' < r$$

when $r' > r$, we have a contribution

$$dA = \frac{\mu_0 r' \omega \rho\, dr' r \sin\theta}{3}, \qquad R > r' > r$$

$$A = \frac{\mu_0 \omega \rho \sin\theta}{3 r^2} \int_0^r dr' (r')^4$$

$$+ \frac{\mu_0 \omega \rho \sin\theta\, r}{3} \int_r^R dr'\, r'$$

$$= \frac{\mu_0 \omega \rho \sin\theta}{3 r^2} \left(\frac{r^5}{5}\right) + \frac{\mu_0 \omega \rho \sin\theta}{3} r \left(\frac{R^2 - r^2}{2}\right)$$

$$\vec{A} = \frac{\mu_0 \omega \rho \sin\theta}{3} r \left(\frac{r^2}{5} + \frac{R^2}{2} - \frac{r^2}{2}\right) \hat{\varphi}$$

$$= \frac{\mu_0 \omega \rho \sin\theta}{3} r \left(\frac{R^2}{2} - \frac{3r^2}{10}\right) \hat{\varphi}$$

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta\, A_\varphi)\, \hat{r}$$

$$- \frac{1}{r} \frac{\partial}{\partial r} (r A_\varphi)\, \hat{\theta}$$

$$= \frac{1}{r \sin\theta} \frac{\mu_0 \omega \rho}{3} r \left(\frac{R^2}{2} - \frac{3r^2}{10}\right) (2 \sin\theta \cos\theta)\, \hat{r}$$

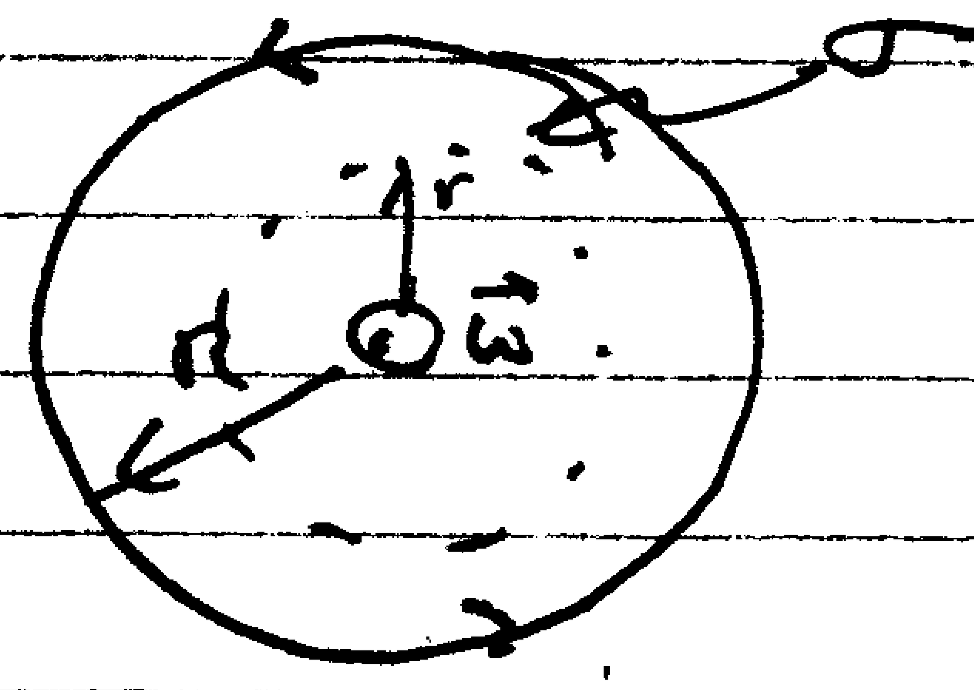
$$- \frac{1}{r} \frac{\mu_0 \omega \rho \sin\theta}{3} \frac{\partial}{\partial r} \left(\frac{r^2 R^2}{2} - \frac{3r^4}{10}\right) \hat{\theta}$$

$$\vec{B} = \frac{\mu_0 \omega \rho (\cos\theta)}{3} \left(R^2 - \frac{3r^2}{5}\right) \hat{r}$$

$$- \frac{\mu_0 \omega \rho \sin\theta}{3} \left(R^2 - \frac{6}{5} r^2\right) \hat{\theta}$$

5.37)

a)



$\vec{m} = I\vec{a}$

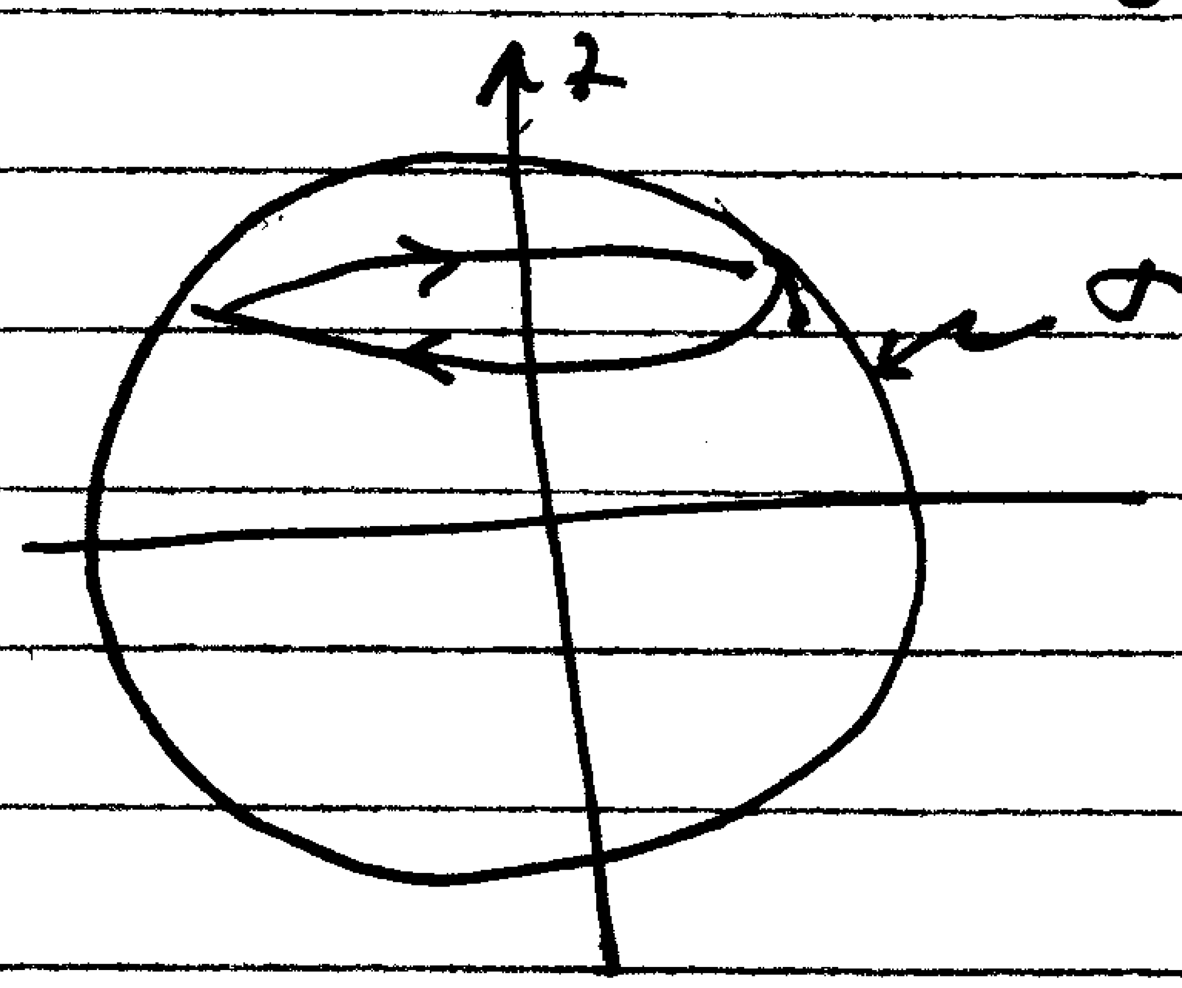
$K = \sigma v \qquad dI = \sigma v\, dr \qquad v = \omega r$

$dm = a\, dI = \pi r^2 dI$

$dm = \pi r^2 \sigma \omega r\, dr$

$m = \pi \sigma \omega \int_0^R r^3 dr = \dfrac{\pi \sigma \omega R^4}{4}, \quad \vec{m} = \dfrac{\pi \sigma R^4}{4} \vec{\omega}$

b)



$area = \pi (R \sin\theta)^2$

$K = \sigma v = \sigma \omega R \sin\theta$

$dI = K\, dl = (\sigma \omega R) R \sin\theta\, d\theta$

$dm = (area)\, dI = \pi (R\sin\theta)^2 (\sigma \omega R) R \sin\theta\, d\theta$

$m = \pi R^4 \sigma \omega \int_0^\pi d\theta \sin^3\theta = \pi R^4 \sigma \omega \left(\frac{4}{3}\right)$

$\vec{m} = \dfrac{4\pi}{3} R^4 \sigma \omega\, \hat{z}$

$\vec{A}_{dipole} = \dfrac{\mu_0}{4\pi} \dfrac{\vec{m} \times \hat{r}}{r^2} = \dfrac{\mu_0}{4\pi} \left(\dfrac{4\pi}{3}\right) R^4 \sigma \omega \dfrac{\hat{\varphi} \sin\theta}{r^2}$

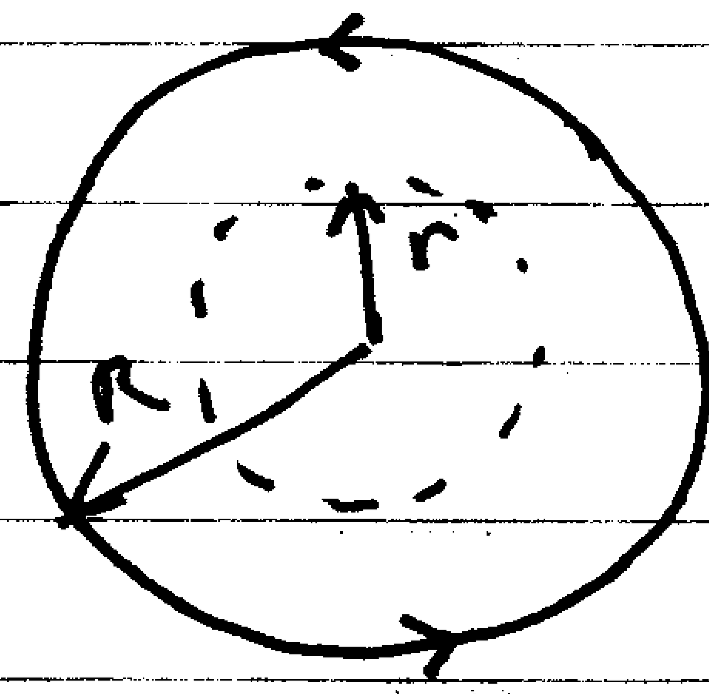
$$\vec{A}_{\text{dipole}} = \frac{\mu_0}{3} R^4 \sigma \omega \frac{\sin\theta}{r^2} \hat{\varphi}$$

which is also the exact result for a spinning sphere
So a spinning sphere of charge / area σ produces only a dipole field - no quadrapole or higher moment fields. (See Equ 5.69)

5-48) For a single loop

$$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$$

For spinning disc



$$dI = \sigma v \, dr \qquad v = \omega r$$

$$dI = \sigma \omega r \, dr$$

$$dB = \frac{\mu_0}{2} \frac{r^2}{(r^2+z^2)^{3/2}} (\sigma \omega r) dr$$

$$B(z) = \frac{\mu_0}{2} \sigma \omega \int_0^R dr \frac{r^3}{(r^2+z^2)^{3/2}}$$

$$\int_0^R dr \frac{r^3}{(r^2+z^2)^{3/2}} = (r^2+z^2)^{1/2} + \frac{z^2}{(r^2+z^2)^{1/2}} \Bigg|_{r=0}^{r=R}$$

$$= (R^2+z^2)^{1/2} + \frac{z^2}{(R^2+z^2)^{1/2}} - z - \frac{z^2}{z}$$

$$= \frac{(R^2+z^2)+z^2}{(R^2+z^2)^{1/2}} - 2z = \frac{R^2+2z^2}{(R^2+z^2)^{1/2}} - 2z$$

$$B(z) = \frac{\mu_0 \sigma \omega}{2} \left[\frac{R^2+2z^2}{(R^2+z^2)^{1/2}} - 2z \right]$$

Now let $z >> R$ and expand the term in brackets

$$\frac{R^2 + 2z^2}{(R^2+z^2)^{1/2}} - 2z = \frac{2z^2\left(1+\frac{R^2}{2z^2}\right)}{z\left(1+\frac{R^2}{z^2}\right)^{1/2}} - 2z$$

$$\left(1+\frac{R^2}{z^2}\right)^{-1/2} = 1 - \frac{R^2}{2z^2} + \frac{3}{8}\left(\frac{R^2}{z^2}\right)^2 + \text{higher order}$$

$$\frac{2z\left(1+\frac{R^2}{2z^2}\right)}{\left(1+\frac{R^2}{z^2}\right)^{1/2}} = 2z\left(1+\frac{R^2}{2z^2}\right)\left(1-\frac{R^2}{2z^2}+\frac{3}{8}\frac{R^4}{z^4}+\cdots\right)$$

$$= 2z\left(1 - \frac{R^2}{2z^2} + \frac{3}{8}\frac{R^4}{z^4} + \frac{R^2}{2z^2} - \frac{R^4}{4z^4} + \cdots\right)$$

$$= 2z\left(1 + \frac{R^4}{8z^4} + \cdots\right)$$

$$B(z) = \frac{\mu_0 \sigma \omega}{2}\left[2z + \frac{R^4}{4z^3} - 2z\right]$$

$$= \frac{\mu_0 \sigma \omega}{2}\frac{R^4}{4z^3} = \frac{\mu_0 \sigma \omega R^4}{8z^3} \quad \text{for } z >> R$$

For the spinning disc problem

we have $\vec{m} = \frac{\pi \sigma R^4}{4} \vec{\omega}$ and a dipole field

$$\vec{B}_{dipole} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

Here $\theta = 0$ and $r \to z$

$$\vec{B}_{dipole} = \frac{\mu_0 m}{4\pi z^3}(2\hat{z}) = \frac{\mu_0 m}{2\pi z^3}\hat{z}$$

Let $m = \frac{\pi \sigma R^4 \omega}{4}$

$$\vec{B}_{dipole} = \frac{\mu_0}{2\pi z^3}\left(\frac{\pi \sigma R^4 \omega}{4}\right)\hat{z}$$

$$= \frac{\mu_0 \sigma \omega R^4}{8 z^3}\hat{z}$$ in agreement with our $z >> R$ approximation